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Identification of Influential Uncertainties in Monte Carlo Analysis

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Monte Carlo simulation is a powerful and a practical tool for evaluating nonlinear systems. Its advantage is that it allows the effects of combinations of uncertainties to be taken into account. When the result of a Monte Carlo simulation is unsatisfactory, further investigations of both the system model and the control system are necessary, and it is important to identify those uncertain parameters that significantly influence the outcome of the simulation. However, the influential parameters are usually difficult to identify because multiple uncertain parameters are incorporated into a simulation simultaneously. A methodology is presented for identifying influential parameters in Monte Carlo analysis. When a Monte Carlo simulation yields an unsatisfactory result, the influential uncertainties are identified by further Monte Carlo simulations incorporating test vectors derived from the original uncertain parameter vector and by a statistical hypothesis test. The method is applied to the simulation results of an unmanned flight system, demonstrating its effectiveness in a practical application.

Nomenclature

H_0 = null hypothesis
M = number of test vectors that incorporate each uncertain parameter in N_{MCSI} test vectors
M_j = number of test vectors that incorporate the jth uncertain parameter in N_{MCSI} test vectors
M_{Fj} = number of unsatisfactory result cases in M_j Monte Carlo simulations
N = number of Monte Carlo simulations
N_{cmb} = number of possible combinations of uncertain parameters in test vector ε_F^*
N_F = number of unsatisfactory result cases in N_{MCSI} simulations
N_{MCE} = number of Monte Carlo simulations for system evaluation
N_{MCSI} = number of Monte Carlo simulations for parameter identification
n = number of uncertain parameters incorporated in Monte Carlo simulation
P_j = upper cumulative probability of $p_j(x)$
$p(x)$, $p_j(x)$ = probability of x failures occurring when the null hypothesis is assumed
p_F = probability of failure in N_{MCSI} Monte Carlo simulations
p_{Fj} = probability of failure in M_j Monte Carlo simulations
p_{success} = probability of mission achievement
r = probability of taking each element of ε_F to create ε_F^*
V_g = ground speed, m/s
X, Y = vehicle position, m
$\dot{Z}$ = sink rate, m/s
z_i = evaluation of ith simulation result (success or failure)
α = level of significance
β_G = side-slip angle of inertial velocity vector, deg
$\Delta Ax_{\mathrm{random}}$ = sensor measurement random noise on x-axis acceleration output
$\Delta Az_{\mathrm{random}}$ = sensor measurement random noise on z-axis acceleration output
ΔC_L = uncertainty of a lift coefficient
ΔC_{L_α} = uncertainty of a lift coefficient derivative with respect to angle of attack, deg^{-1}
ΔC_{m_q} = uncertainty of a pitching moment coefficient derivative with respect to pitch rate, s/deg
ΔC_{m_α} = uncertainty of a pitching moment coefficient with respect to angle of attack, deg^{-1}
$\Delta \dot{X}_{\mathrm{nav}}$ = initial navigation error of X-direction velocity, m/s
ε = uncertain parameter vector
ε_F = uncertain parameter vector that yields unsatisfactory simulation result
ε_F^* = test vector created from ε_F
Θ = pitch angle, deg
σ = standard deviation
Φ = roll angle, deg
Ψ = yaw angle, deg

Introduction

IN the development of autonomous aerospace vehicles, particularly those such as aerospace planes that are required to operate in extreme flight regimes, opportunities for testing vehicle performance in a real flight environment are often limited. However, because the system must have sufficient performance to be able to complete its mission, extensive preflight analysis becomes a critically important part of the development process.

Monte Carlo simulation applied to system evaluation [Monte Carlo evaluation (MCE)] is a powerful tool for preflight system performance analysis.[1–3] Although requiring great computational resources, its results are quite rewarding, and Monte Carlo simulation is becoming ever more practical as the power of modern computers increases. MCE has several advantages: nonlinear systems can be evaluated directly, uncertain parameters can be assumed as physical values, and the Monte Carlo results reflect the influences of various combinations of uncertain parameters.

The validity of MCE has been established for linear time-invariant systems[4–6] and then has been applied to the development of an

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unmanned flight system, where simulation results were found to agree with those of actual flight experiments.[7] Furthermore, the application of MCE has been extended to control system design, for the optimization of control parameters in a nonlinear system. Here, the control parameters are optimized within a fixed control structure, while Monte Carlo simulation evaluates the system and is used to derive a cost function. A number of algorithms have been used for the stochastic optimization, such as genetic algorithms,[8–10] the mean tracking method,[11,12] and simulated annealing.[13] Because large numbers of simulation trials are required, a parallel processing system has been developed to reduce the computation time.[12,14]

Because all uncertain parameters are incorporated randomly and simultaneously into each simulation trial according to their assumed probability distributions, the influences of multiple uncertainties on the system can be evaluated. When a simulation trial yields an unsatisfactory result, it is important to identify the significant influential uncertain parameters. Once these parameters are identified, the system design can be reconsidered, for example, further ground testing might be conducted to reduce the magnitude of the uncertain parameters, or the control system might be redesigned to increase its robustness against influential uncertain parameters. However, because a large number of uncertain parameters may be incorporated simultaneously into the simulation, it can be difficult to identify those that are the most influential.

This paper presents a methodology for identifying influential uncertain parameters by means of further Monte Carlo simulations incorporating test vectors and by application of a statistical hypothesis test. Test vectors are created from an uncertain parameter input vector that causes an unsatisfactory simulation result. These test vectors are used as inputs to additional Monte Carlo simulations for parameter identification [Monte Carlo simulation for identification (MCSI)]. Hypothesis testing is then applied to the result of MCSI to identify the influential uncertain parameters. To demonstrate the validity of this identification method, it is applied to the MCE result of an unmanned autonomous flight system. It is demonstrated that influential parameters are identified and that the identification method is effective for the development of flight vehicles.

Monte Carlo Simulation

Monte Carlo simulation investigates the influences of various uncertainties on a flight system using random sampling of uncertain parameters.[15] A flight system is analyzed, and its control system is designed based on mathematical models. However, in the real world many uncertain parameters exist such as uncertainties in aerodynamics, vehicle model, sensor measurement, actuator dynamics, initial conditions, and environmental conditions. Investigating the influences of these uncertain parameters on the system is necessary in preflight analysis so that the system satisfies its design requirements in spite of these uncertainties.

The Monte Carlo simulation process is shown in Fig. 1. After an input uncertainty vector ε_i, whose elements are generated randomly, is incorporated into a system model, a flight simulation trial is performed and a result z_i is obtained. If the result meets the requirement (success),

$$z_i = 0 \tag{1a}$$

otherwise (failure),

$$z_i = 1 \tag{1b}$$

A new ε_i is then generated, and the process repeats for N iterations. The final result is obtained either as a probability of mission achievement,

$$p_{\text{success}} = 1 - \frac{\sum_i z_i}{N} \tag{2}$$

or as a distribution of target state variables.

This paper introduces two types of Monte Carlo simulation: MCE, which is used for system evaluation, and MCSI, which is used for identifying influential uncertain parameters. MCE assesses a system's robustness against uncertainties. All uncertain parameters are generated randomly and simultaneously according to their assumed distributions, and the system is, thus, evaluated against different combinations of uncertain parameters. Furthermore, nonlinear systems can be evaluated directly. On the other hand, MCSI identifies the uncertain parameters that are influential on the simulation outcome. Some input vectors to MCE simulations will give results that fail to meet the requirements, and the elements of such ε_i vectors are used to create test vectors ε_F^*, which is described in the next section. Influential uncertain parameters are investigated by further Monte Carlo simulations that incorporate ε_F^*.

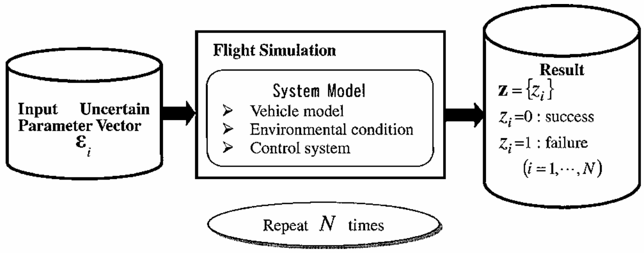

Fig. 1 Monte Carlo simulation.

Identification of Influential Uncertainties

The uncertain parameter identification method presented in this paper consists of two main steps. In the first step, MCSI, Monte Carlo simulations incorporating test vectors are performed. These test vectors are generated using elements of the uncertain parameter vectors that give unsatisfactory simulation results. The generation of these test vectors is a key technique of the first step. In the second step, hypothesis testing is applied to the results of MCSI to identify the influential uncertain parameters. In the discussion that follows, the generation of test vectors and hypothesis testing are described in turn, and the identification procedure is then summarized. Finally, a modified technique for generating test vectors is presented.

Test Vector Generation

In the identification method, a test vector ε_F^* is created from an ε_F that yields an unsatisfactory simulation result. Monte Carlo simulations incorporating different ε_F^* then investigate randomly the influence of each uncertain parameter $\varepsilon_F(j)$ on the outcome of a simulation. Figure 2 shows the process of generating ε_F^*.

Each test vector ε_F^* is a subset of the original uncertainty vector ε_F. To generate a test vector, each element of ε_F is selected for incorporation into ε_F^* with a probability r. Namely,

if $U(0, 1) \leqq r$

$$\varepsilon_F^*(j) = \varepsilon_F(j)$$

else

$$\varepsilon_F^*(j) = 0, \qquad j = 1, \ldots, n \tag{3}$$

where $U(0, 1)$ is a random variable from a uniform distribution between 0 and 1. A total of N_{MCSI} vectors of ε_F^* are generated in this manner.

The objective of generating test vectors is to create various combinations of uncertain parameters using the elements of ε_F, and so r should be chosen so that the number of combinations is maximized. Because the average number of uncertain parameters incorporated in any single test vector is rn, the possible number of combinations is the ways of selecting rn from a total number of uncertain parameters n. Thus,

$$N_{\text{cmb}} = {}_nC_{rn} \tag{4}$$

Because N_{cmb} is expressed as a binomial coefficient, it has a maximum value at

$$rn = \tfrac{1}{2}n \tag{5}$$

therefore,

$$r = 0.5 \tag{6}$$

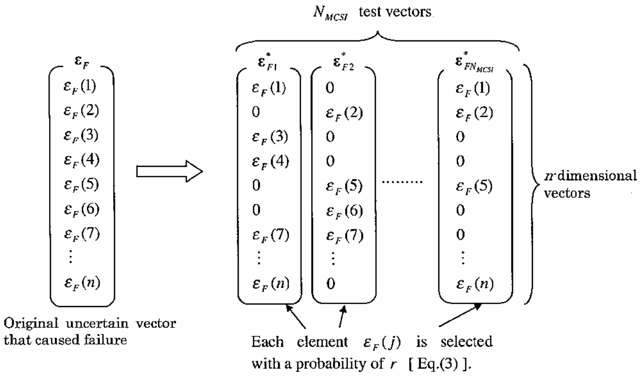

Fig. 2 Generation of test vectors.

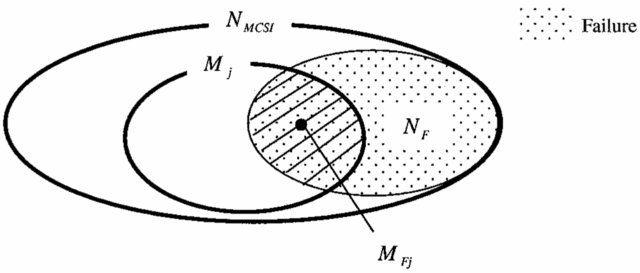

Fig. 3 Relationship between numbers of simulations and failures.

For this reason $r = 0.5$ is recommended, although other values, such as $r = 0.4$ or 0.6, may also be acceptable.

MCSI is then performed for each of the N_{MCSI} vectors of ε_F^*. If there are N_F unsatisfactory outcomes out of N_{MCSI} simulation trials, the probability of an unsatisfactory result is

$$p_F = N_F / N_{\mathrm{MCSI}} \tag{7}$$

If the parameter $\varepsilon_F(j)$ is influential, the simulation result is more likely to be unsatisfactory when the input vector ε_F^* incorporates $\varepsilon_F(j)$ than when it does not. Let the number of vectors ε_F^* that incorporate the element $\varepsilon_F(j)$ be M_j and the number of unsatisfactory outcomes out of the M_j simulations that incorporate the element be M_{Fj}. The relationship between these quantities is shown in Fig. 3. The probability of unsatisfactory simulation trial that incorporates $\varepsilon_F(j)$ is

$$p_{Fj} = M_{Fj} / M_j \tag{8}$$

If $\varepsilon_F(j)$ has a significant influence such as to cause an unsatisfactory outcome,

$$p_{Fj} > p_F \tag{9}$$

Hypothesis testing, described in the next section, is used to decide whether p_{Fj} is significantly greater than p_F.

The advantage of this method is that influential parameters can be identified even when failure is caused by a combination of more than one uncertain parameter. For example, when an unsatisfactory result is caused by the combination of jth and kth uncertain parameters, both M_{Fj} and M_{Fk} become greater because each uncertain parameter contributes to the failure, which means that both p_{Fj} and p_{Fk} also become greater than p_F. Thus, influential uncertain parameters can be detected even when failure is caused by a particular combination of multiple uncertain parameters.

Hypothesis Testing

The other main step of the identification procedure is the application of hypothesis testing to the results of the MCSI. Hypothesis testing is the process of inferring from a sample whether or not a given statement about the population (a hypothesis) appears to be true.[16] In hypothesis testing, a null hypothesis H_0 is assumed to be tested. If the data in the sample strongly disagree with the null hypothesis, the null hypothesis is rejected and the conclusion is the negation of the null hypothesis. For our problem, the null hypothesis H_0 is each uncertain parameter has no influence on whether the result of MCSI is satisfactory.

Because the obtained quantity M_{Fj} is expected to become large when the jth uncertain parameter is actually influential, an upper-tailed test[16] should be conducted. Under the null hypothesis, the probability of M_{Fj} or more failures occurring among M_j simulations is calculated for each uncertain parameter. If the calculated probability is significantly small, the null hypothesis is rejected and the jth parameter is determined to be influential.

Under the null hypothesis, the probability of x failures occurring, $p_j(x)$, is calculated as

$$p_j(x) = \frac{\left({}_{N_F}C_x\right)\left({}_{N_{\mathrm{MCSI}}-N_F}C_{M_j-x}\right)}{{}_{N_{\mathrm{MCSI}}}C_{M_j}}$$

$$x = 0, \ldots, x_{\max}, \qquad x_{\max} = \min\{M_j, N_F\} \tag{10}$$

where ${}_{N_{\mathrm{MCSI}}}C_{M_j}$ is the number of ways of selecting M_j of jth uncertain parameter from a total number of simulations N_{MCSI}, ${}_{N_F}C_x$ is the number of ways of selecting x failures from a total of N_F failures, and ${}_{N_{\mathrm{MCSI}}-N_F}C_{M_j-x}$ is the number of ways of selecting $(M_j - x)$ successes from a total of $(N_{\mathrm{MCSI}} - N_F)$ successes. The operator min in Eq. (10) indicates a minimum value. The distribution expressed by Eq. (10) is called a hypergeometric distribution[16] and is shown in Fig. 4. Because x is a discrete value, the distribution of $p_j(x)$ is of the discrete type. As x has a value from 0 to $x_{\max}$,

$$\sum_{x=0}^{x_{\max}} p_j(x) = 1 \tag{11}$$

The probability that the failure x occurs the same number of times as, or more often than, the experimental result M_{Fj} is given by

$$P_j = \Pr[x \geq M_{Fj}] = \sum_{x=M_{Fj}}^{x_{\max}} p_j(x) \tag{12}$$

where $\Pr[\cdot]$ indicates a probability. P_j is called the p value, that is, the smallest significance level at which the null hypothesis would be rejected for the obtained M_{Fj} (Ref. 16).

A very small value of P_j means that the result M_{Fj} is unlikely to occur under the null hypothesis. In this case, the null hypothesis should be suspected as being true because M_{Fj} actually occurred in spite of a low probability of so doing. Thus, the decision rule is that when P_j is smaller than the level of significance α, which is determined by the practitioner, the null hypothesis is rejected; in other words, the jth uncertainty is determined to be an influential uncertain parameter.

The level of significance α, which indicates the risk of the practitioner in accepting a false result, should be determined depending on the applied system. Generally, α is often set to 5 or 1% (Ref. 16). However, because the number of obtained P_j is n, $P_j = 1/n$ is likely to occur by chance. As the number of uncertain parameters n

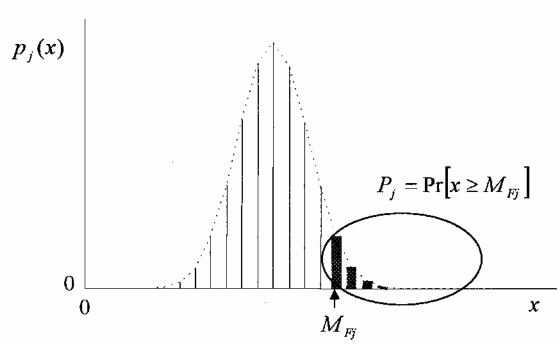

Fig. 4 Probability distribution of x failures occurring.

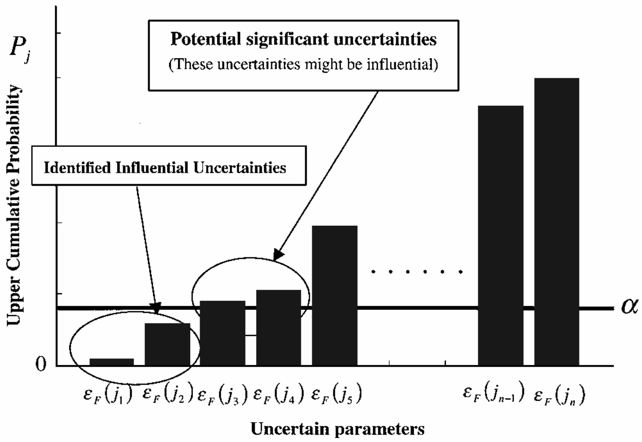

Fig. 5 Ranking the upper cumulative probabilities in ascending order.

increases, a small value of P_j is likely to occur by chance, and so when n is large, $\alpha = 5$ or 1% may be inappropriate. In this case, it would be better to select a value of α, at most, as one order of magnitude smaller than the probability $1/n$. Namely,

$$\alpha = 1/(10n) \tag{13}$$

However, α should be chosen smaller than the value given by Eq. (13) when high reliability is required for the hypothesis test. Basically, the value of α must be chosen by the practitioner according to the application.

The value P_j, $j = 1, \ldots, n$, itself is more important. Because P_j indicates a probability of no influence on the failure, the possibility of jth uncertain parameter's contribution to the failure becomes greater as P_j becomes smaller. The jth uncertain parameter may be influential when the value of P_j is greater than, but close to, α. Potential influential uncertainties are found by ranking the values P_j in ascending order, as shown in Fig. 5. To confirm whether the potential influential uncertainties are indeed influential, further MCSI should be performed. If the jth uncertain parameter is actually influential, the corresponding P_j becomes small as N_{MCSI} increases. Thus, the value and order of P_j are essential for the identification of influential uncertain parameters.

Summary of the Identification Procedure

The procedure of the presented identification method is summarized in Fig. 6. After a system is evaluated by MCE, the identification procedure is carried out by the following steps. First, take an uncertain parameter input vector ε_F that results in a failure in MCE. Second, generate test vectors, ε^*_{Fi}, $i = 1, \ldots, N_{\mathrm{MCSI}}$, using the elements of ε_F. When ε^*_{Fi} is generated, the number of times each uncertainty is incorporated in N_{MCSI} test vectors M_j is obtained. Third, perform Monte Carlo simulations incorporating ε^*_{Fi} (MCSI). Fourth, count both the number of unsatisfactory simulation results, N_F, in N_{MCSI} simulations and the number of each uncertain parameter M_{Fj}, $j = 1, \ldots, n$, which are included N_F test vectors that caused unsatisfactory simulation results.

In the next step, hypothesis testing is carried out as follows. Fifth, assume that each uncertain parameter has no influence on the simulation result (the null hypothesis). For each uncertain parameter $\varepsilon_F(j)$, calculate $p_j(x)$, the probability of x failures occurring among M_j simulations. Sixth, for each uncertain parameter, calculate the p value $P_j = \Pr[x \geq M_{Fj}]$. Seventh, rank P_j in ascending order to see the significance of each uncertain parameter contributing to an unsatisfactory result. Finally, identify influential uncertain parameters by comparing P_j with the level of significance α. If P_j is smaller than α, the null hypothesis is rejected and the corresponding uncertain parameter $\varepsilon_F(j)$ is determined to be influential. When P_j is a little greater than α (potential significant uncertain parameter), additional MCSI may be necessary to confirm the significance of uncertain parameter $\varepsilon_F(j)$.

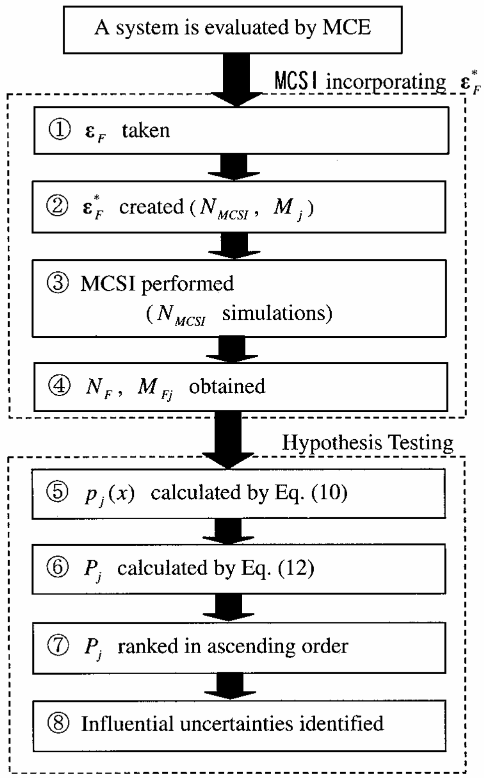

Fig. 6 Procedure of influential uncertain parameter identification.

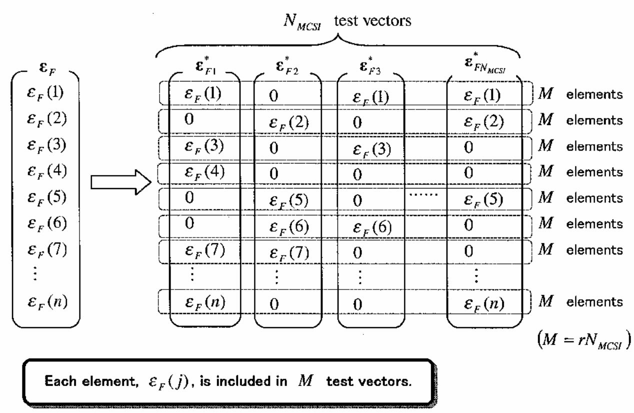

Fig. 7 Modified test vector generation.

Modification of Test Vector Generation

In this section, a modified method for test vector generation is introduced. An advantage of this method is that the order of P_j is obtained without calculating P_j; in other words, the candidates of influential uncertainty, which have small values of P_j, are found before hypothesis testing. Another advantage is that computation load of hypothesis testing is reduced. Test vectors are generated by the modified method as follows.

Test vectors are generated so that the number of times each uncertain parameter occurs in the test vector set, M_j, is the same for all uncertain parameters. That is,

$$M_j = M, \qquad j = 1, \ldots, n \tag{14}$$

where M is chosen to be

$$M = rN_{\mathrm{MCSI}} \tag{15}$$

The number M is rounded off to the nearest integer. This manner of generation is shown in Fig. 7. Each $\varepsilon_F(j)$ is allocated randomly to M test vectors. When Eq. (14) is satisfied, Eq. (10) becomes

$$p_j(x) = p(x) = \frac{\binom{N_F}{C_x}\binom{N_{\mathrm{MCSI}} - N_F}{C_{M-x}}}{\binom{N_{\mathrm{MCSI}}}{C_M}}, \qquad j = 1, \ldots, n \tag{16}$$

The distribution $p_j(x)$ then becomes the same for all uncertain parameters $\varepsilon_F(j)$. The upper cumulative probability P_j is calculated using Eq. (12) as

$$P_j = \sum_{x = M_{Fj}}^{x_{\max}} p(x), \qquad j = 1, \ldots, n \tag{17}$$

An advantage of this method is that the value of P_j in Eq. (17) depends only on the number of $p(x)$ to be summed. Because P_j becomes smaller as M_{Fj} becomes greater, the ascending order of P_j is consistent with the descending order of M_{Fj} that is obtained by MCSI. Thus, the order of P_j, which is the order of parameter's significance, can be obtained before calculating $p(x)$ or P_j. Another advantage is that, because the calculation of $p_j(x)$ for all j is unnecessary, the computation load of hypothesis testing is reduced. This reduction of computation load becomes increasingly effective as N_{MCSI} becomes larger because $p(x)$ requires a calculation of three binomial coefficients in Eq. (16). However, in the modified procedure, the number M must be determined before the generation of test vectors, whereas the generation procedure shown in Fig. 2 has no such a restriction. The generation procedure shown in Fig. 2 is simpler than the modified procedure. Thus, in practice the generation procedure of Fig. 2 may be convenient in some applications although the modified procedure has the advantages described earlier.

Application to Autonomous Flight System

To demonstrate the practical application of the identification method, it is now applied to the identification of influential uncertainties in the automatic landing flight experiment (ALFLEX) unmanned autonomous flight system. First, the ALFLEX flight-test program and its Monte Carlo simulation results are introduced. Next, influential uncertain parameters are identified by the procedure described. Then, the influence of N_{MCSI} on the reliability of the identification result is investigated. Finally, the identification method is compared with sensitivity analysis, a conventional method for identifying influential parameters. The advantages of the presented identification method are also discussed.

ALFLEX Program

The ALFLEX program was conducted in 1996 to establish automatic landing technology for a future Japanese unmanned reentry vehicle. There were 13 automatic landing flight experiments successfully conducted at Woomera Airfield in South Australia. A three-view schema of the ALFLEX vehicle is shown in Fig. 8. Details of the ALFLEX experiment and its control system are described in Ref. 17.

Figure 9 shows the landing experiment flight profile. The ALFLEX vehicle starts the experiment suspended from a helicopter. After the vehicle is released into free flight about 2700 m from the runway threshold at a height of 1500 m, it is guided to follow a predetermined reference path, controlled by an onboard computer. The flight is divided into a number of phases: an equilibrium glide phase, a preflare phase, and a final flare phase. In the preflare phase, the vehicle pitches up and air speed is reduced; then in the final flare phase, sink rate becomes the primary controlled variable to give a soft landing. Different longitudinal guidance laws were designed for each flight phase based on corresponding linear models and were smoothly connected. The system is, thus, highly nonlinear, and is, therefore, a case for which MCE is effective.

In preflight Monte Carlo analysis, various uncertain parameters and their distributions were assumed. The types of uncertainties are shown in Fig. 10, and the total number of uncertain parameters considered was $n = 103$, as shown in Table 1. Computer simulations incorporating the uncertainties were performed of flights from vehicle release to landing, and the simulation outcomes were classified according to the touchdown state requirements shown in Table 2. Table 3 shows the numbers of simulation cases in which one or more of the requirement criteria were exceeded. In Table 3, unstable means that the vehicle became unstable and was unable to reach the runway. The total number of unsatisfactory cases is shown at the bottom of Table 3, and its value differs from a simple sum of the numbers of requirement violations shown because in some cases

Table 1 Uncertain parameters for ALFLEX model

Category	Number of parameters
Sensor error	38
Vehicle model error	41
Environmental condition	6
Initial condition	18
Total	103

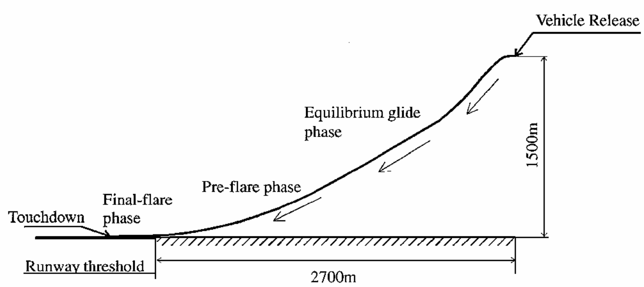

Fig. 9 Flight profile of ALFLEX landing experiment.

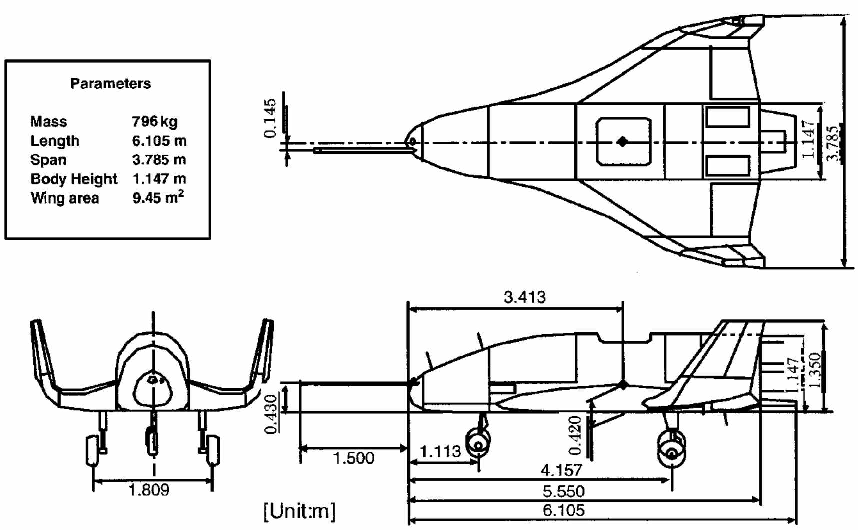

Fig. 8 ALFLEX vehicle.

Table 2 ALFLEX touchdown performance requirements

Touchdown state	Requirement
Position[a]	$X > 0$ m, $\lvert Y\rvert < 18$ m
Velocity	$V_G < 62$ m/s, $\dot{Z} < 3$ m/s
Attitude	$\Theta < 23$ deg, $\lvert\Phi\rvert < 10$ deg, $\lvert\Psi\rvert < 8$ deg
Side slip	$\lvert\beta_G\rvert < 8$ deg

[a] In runway coordinate system; the origin is at the runway threshold, the X axis is directed along the runway centerline and the Z axis is directed downward.

Table 3 Numbers of cases exceeding each requirement in 1000 MCE

Parameter	Number of cases
X	7
Y	0
$\dot{Z}$	31
V_G	8
Θ	0
Φ	0
Ψ	3
β_G	0
Unstable	1
Total	40

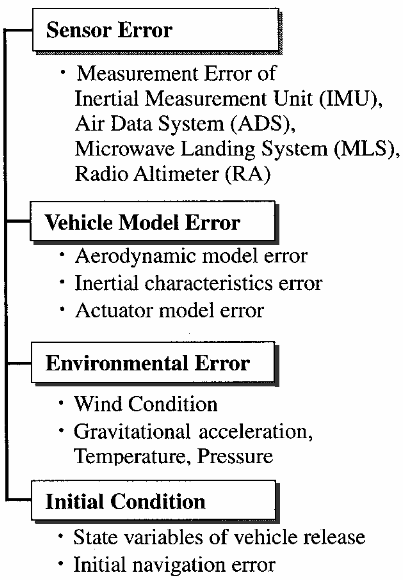

Fig. 10 Uncertain parameters incorporated into ALFLEX Monte Carlo simulation.

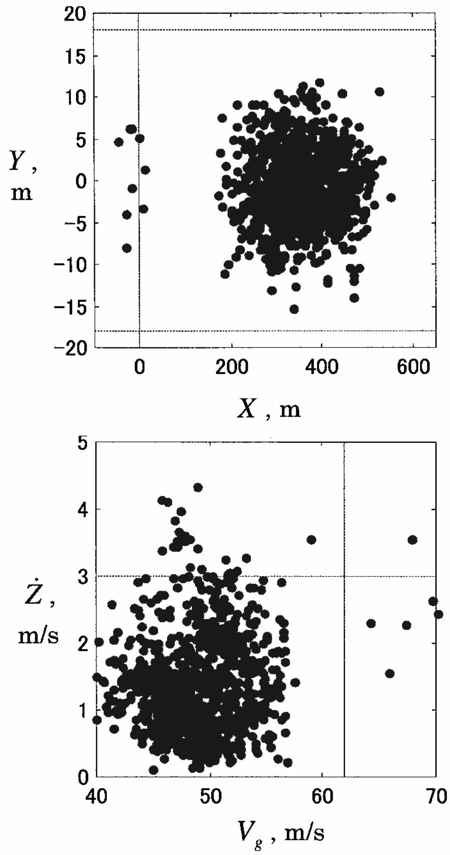

Fig. 11 Touchdown states of 1000 MCE.

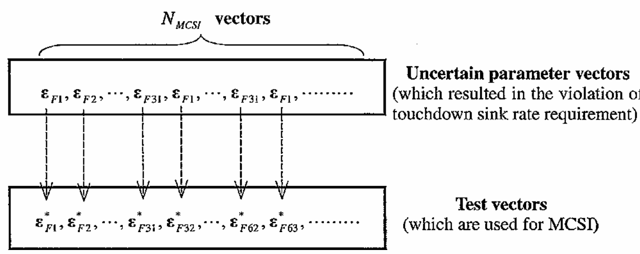

Fig. 12 Test vector generation for the ALFLEX example.

more than one requirement was exceeded. In this result, 4% of all cases were unsatisfactory. An example of the distribution of simulation touchdown states is shown in Fig. 11. The solid lines in these graphs indicate the requirements of Table 2.

Identification of Influential Uncertainties

From the results of 1000 MCE ($N_{\mathrm{MCE}} = 1000$) as shown in Table 3, the touchdown sink rate requirement was found to be the most frequently violated touchdown state with 31 cases failing to meet the requirement. The influential uncertain parameters affecting touchdown sink rate will be now identified.

In this example, the number of uncertain vectors ε_F that caused unsatisfactory simulation results is 31. If one of the 31 uncertain vectors is selected and used to generate test vectors according to the procedure shown in Fig. 2 or 7, then the influential uncertain parameters for the selected vector will be identified. Thus, a set of influential parameters is derived for each of the 31 uncertain vectors. However, because the identification procedure, which is shown in Fig. 6, must be repeated 31 times for each ε_F, substantial computation time is necessary. To avoid this burden in this example, the 31 uncertain vectors of ε_F are treated simultaneously to derive the influential uncertain parameters for all 31 unsatisfactory cases. In practice, it is important to find the influential uncertain parameters that affect the most unsatisfactory cases.

The identification procedure is as follows. First, take the 31 uncertain parameter vectors of simulation cases that failed to meet the touchdown sink rate requirement. Second, arrange these 31 vectors ε_{Fi} in succession. Line up N_{MCSI} uncertain parameter vectors, ε_{Fi}, $i = 1, \ldots, 31$, in sequence as shown in Fig. 12, repeating the sequence as necessary because N_{MCSI} is usually larger than 31. Third, create N_{MCSI} test vectors ε_F^* from the corresponding N_{MCSI} vectors of ε_F, such that Eqs. (14) and (15) are satisfied, as shown in Fig. 7. Fourth, conduct Monte Carlo simulations incorporating these test vectors ε_{Fi}^*, $i = 1, \ldots, N_{\mathrm{MCSI}}$. Fifth, obtain the number of unsatisfactory simulation results, N_F, and the quantities M_{Fj} for each uncertain parameter $\varepsilon_F(j)$, $j = 1, \ldots, n$. Finally, calculate P_j for each uncertain parameter using Eqs. (16) and (17). When P_j is less than the level of significance α, the jth uncertain parameter is determined to be influential.

In this analysis, the constants r and N_{MCSI} are set to

$$r = 0.5 \tag{18a}$$

Table 4 Identified influential uncertain parameters

Uncertain parameter	P_j, %	M_{Fj}	M
ΔC_{m_α}	4.70×10^{-13}	108	500
$\Delta Ax_{\text{random}}$	5.37×10^{-4}	91	500
$\Delta Az_{\text{random}}$	1.27×10^{-3}	90	500

Table 5 Results of 1000 MCE

Case	Removed uncertain parameter	No. of failures[a]
1	None	31
2	ΔCm_α, $\Delta Ax_{\text{random}}$, $\Delta Az_{\text{random}}$	0
3	ΔCm_α	20
4	$\Delta Ax_{\text{random}}$	29
5	$\Delta Az_{\text{random}}$	26
6	ΔCm_α, $\Delta Az_{\text{random}}$	6
7	ΔCm_α, $\Delta Ax_{\text{random}}$	7
8	$\Delta Ax_{\text{random}}$, $\Delta Az_{\text{random}}$	17

[a]Number of cases exceeding touchdown sink rate requirement.

$$N_{\text{MCSI}} = 1000 \tag{18b}$$

Therefore,

$$M = rN_{\text{MCSI}} = 500 \tag{18c}$$

The level of significance is obtained using Eq. (13). Namely,

$$\alpha = 1/(10 \times 103) \approx 0.1\% \tag{18d}$$

As a result of N_{MCSI} Monte Carlo simulations, the number of unsatisfactory simulation cases is

$$N_F = 134 \tag{18e}$$

The identified uncertain parameters and the corresponding values of M_{Fj} and P_j are shown in Table 4. The uncertain parameters are an aerodynamic uncertainty ΔC_{m_α} and two sensor measurement random noises $\Delta Ax_{\text{random}}$ and $\Delta Az_{\text{random}}$. The values of P_j for these three uncertainties are much less than α, indicating that these parameters affect touchdown sink rate significantly, whereas P_j for the other 100 uncertain parameters are greater than α. In this example, because M is chosen to be the same value for all uncertain parameters, the ascending order of P_j is consistent with the descending order of M_{Fj}.

To confirm that the identified uncertainties are indeed influential, the three identified parameters are removed from the original N_{MCE} vectors of ε and Monte Carlo evaluation is performed again. The result, shown in Fig. 13, can be compared with Fig. 11, which is the result before the removal of the identified parameters from the simulations. The numbers of cases now violating touchdown sink rate requirements are shown in Table 5. When the three identified uncertain parameters are omitted from the Monte Carlo simulations, the number of resulting unsatisfactory cases becomes zero (case number 2). On the other hand, if any one of the identified uncertain parameters is incorporated, cases that violate the requirement occur (cases 3–8). These results confirm that the three identified uncertainties are indeed influential on touchdown sink rate.

Table 5 also shows that the number of unsatisfactory cases arising when ΔC_{m_α} is removed from the simulations is much less than for simulations incorporating ΔC_{m_α}. It is, therefore, considered that ΔC_{m_α} is the most influential parameter on touchdown sink rate. This result is consistent with Table 4 in which the p value of ΔC_{m_α} is significantly smaller than for the other uncertain parameters.

Influence of N_{MCSI}

Generally, the result of a hypothesis test becomes more reliable as the number of samples increases. The influence of N_{MCSI} on the result of the hypothesis test is now investigated.

In Fig. 14, the values of P_j for each uncertain parameter are plotted for several values of N_{MCSI}. The values of P_j for all uncertain

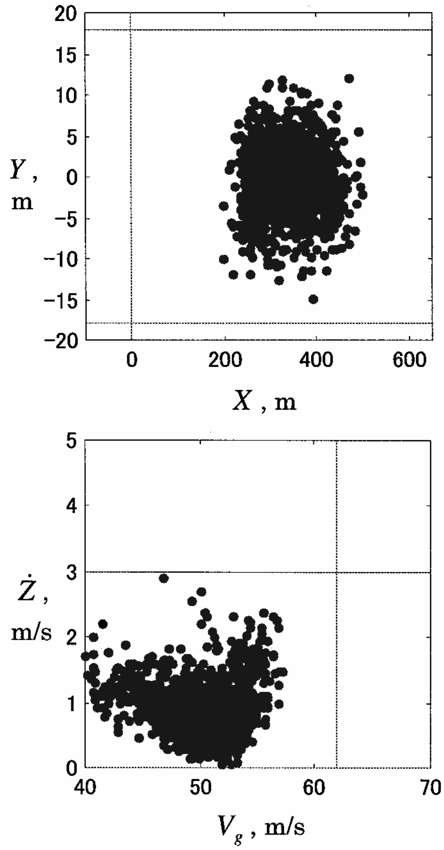

Fig. 13 Touchdown states of 1000 MCE excluding uncertainties of ΔC_{m_α}, $\Delta Ax_{\text{random}}$, $\Delta Az_{\text{random}}$.

parameters $\varepsilon_F(j)$, $j = 1, \ldots, n$, are indicated by dots. The three values of P_j that correspond to the identified three influential uncertain parameters, which are shown as dotted lines, become small as N_{MCSI} increases, whereas the P_j values for most uncertainties are greater than α. Thus, influential uncertain parameters are more likely to be identified as N_{MCSI} increases. In this example, when N_{MCSI} is greater than 800, the values of P_j for ΔC_{m_α}, $\Delta Ax_{\text{random}}$, and $\Delta Az_{\text{random}}$ are less than the level of significance. On the other hand, identification becomes more difficult as N_{MCSI} becomes smaller. For example, when $N_{\text{MCSI}} = 200$, because the values of P_j for $\Delta Ax_{\text{random}}$ and $\Delta Az_{\text{random}}$ are greater than α, the null hypothesis is unable to be rejected. However, ΔC_{m_α} can be identified as an influential parameter even when $N_{\text{MCSI}} = 200$ because P_j for ΔC_{m_α} is much less than α. From the results, when the influence of an uncertain parameter on the simulation outcome is strong, the uncertain parameter can be identified even when N_{MCSI} is small, indicating that the value P_j shows the significance of each uncertain parameter.

Comparison with Sensitivity Analysis

The presented identification method is now compared with sensitivity analysis, which is a conventional method for finding influential uncertain parameters. After influential uncertainties on touchdown sink rate are analyzed by sensitivity analysis, the advantages of the presented identification method are discussed.

In the sensitivity analysis, the influence of each uncertain parameter is individually investigated by simulation; in other words, only one uncertain parameter is considered at a time. When a significant failure outcome is due to the incorporation of an uncertain parameter, the parameter is identified as influential. For each simulation in the ALFLEX sensitivity analysis, the 3σ value of the uncertain parameter under investigation was incorporated and the touchdown sink rate result expressed as a difference from the nominal value of 0.61 m/s. Figure 15 shows the 10 most influential uncertain parameters on touchdown sink rate as determined by this process. By this method, ΔC_{m_α} is identified as an influential uncertain parameter because the touchdown sink rate for the case incorporating the 3σ value of ΔC_{m_α} is 3.47 m/s ($= 0.61 + 2.86$), which exceeds the

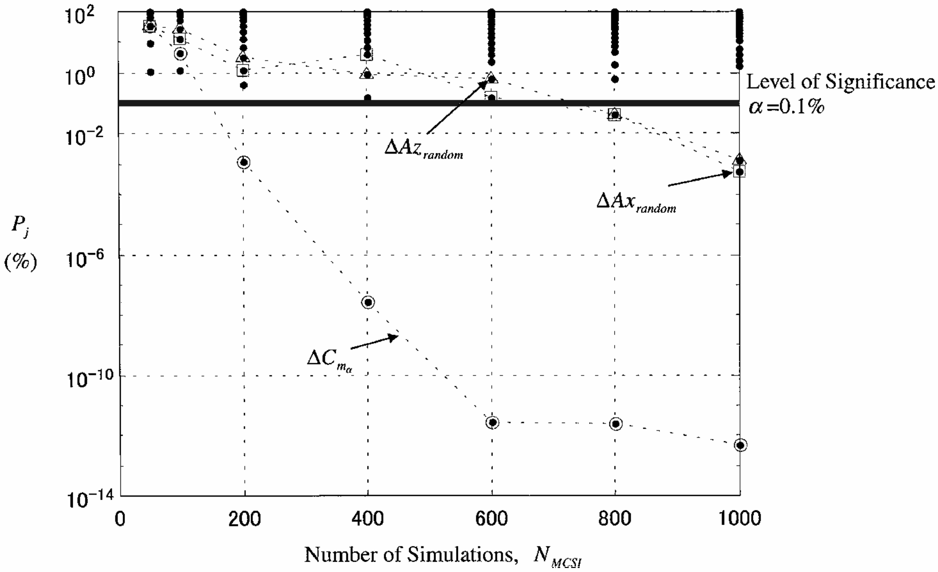

Fig. 14 Influence of number of simulations N_{MCSI} on the hypothesis test.

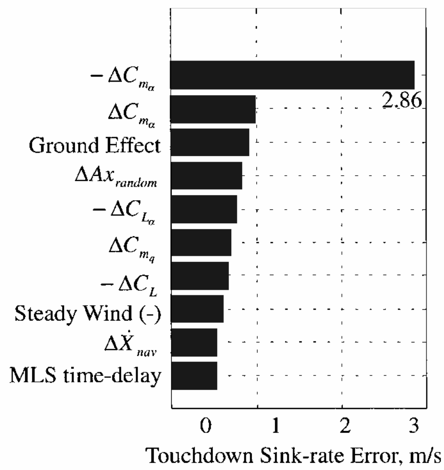

Fig. 15 Influential uncertain parameters on touchdown sink rate by sensitivity analysis.

requirement of 3.0 m/s. Because such a result is of great consequence for an automatic landing experiment, a preliminary flight test of the actual ALFLEX vehicle was conducted to estimate its aerodynamic characteristics and the variation of ΔC_{m_α} was consequently reduced.[11]

Although ΔC_{m_α} is correctly identified as an influential uncertain parameter, sensitivity analysis fails to identify $\Delta Ax_{\mathrm{random}}$ and $\Delta Az_{\mathrm{random}}$. The influence of $\Delta Ax_{\mathrm{random}}$ is similar to those of other parameters except ΔC_{m_α} in Fig. 15, and $\Delta Az_{\mathrm{random}}$ was unable to be identified. From the results, sensitivity analysis is insufficient to identify influential parameters when the following cases occur in MCE. First, failure cases might occur due to particular combinations of uncertain parameters, even though the sensitivity of the system to each of these parameters individually may be quite small. In such cases, sensitivity analysis is unable to reveal the influential uncertain parameters because it only examines the influence of each uncertain parameter individually. Second, the influences of random noises, such as $\Delta Ax_{\mathrm{random}}$ or $\Delta Az_{\mathrm{random}}$, can vary depending on their time histories. For these uncertain parameters, sensitivity analysis can be ineffective because only a single simulation is performed for each uncertainty. In MCE, because multiple simulations incorporating uncertain random noises are carried out using different random generation seeds (and, thus, different time histories) for each trial, such sensitivities are revealed. To investigate the influence of random noises, 1000 MCE are performed incorporating each random noise, $\Delta Ax_{\mathrm{random}}$ or $\Delta Az_{\mathrm{random}}$, as shown in Fig. 16. The touchdown sink rates vary from less than 0.1 to 2.2 m/s when $\Delta Az_{\mathrm{random}}$ alone is incorporated. The sink rate of 2.2 m/s is quite significant result for one parameter's influence, whereas the influence of 0.1 m/s is almost negligible. Thus, it is shown that touchdown performance is sometimes seriously affected by the difference of random seeds between simulations. If local characteristics of time sequence random data, such as mean value, are biased during a critical flight phase, the vehicle's landing performance is seriously affected.

In summary, although sensitivity analysis gives valuable information about influential uncertain parameters, there are cases where it will fail to detect influential parameters. The identification method presented in this paper overcomes these problems because the influence of each uncertain parameter is randomly investigated using Monte Carlo simulation, MCSI.

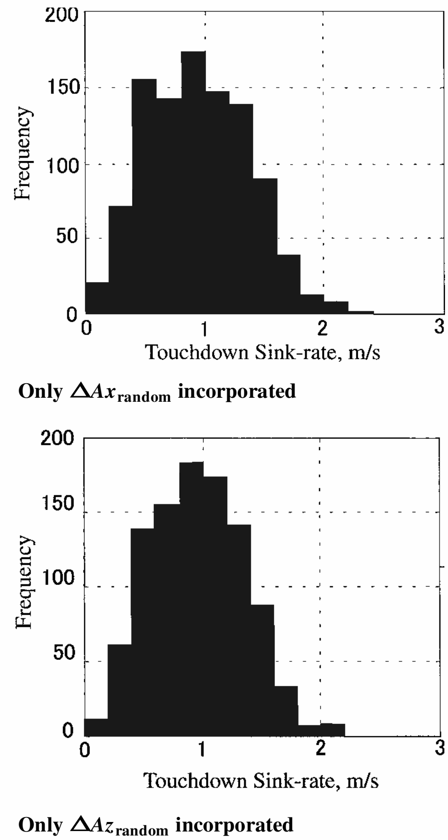

Fig. 16 Influence of random seeds on touchdown sink rate in 1000 MCE.

Conclusions

A method of identifying uncertain parameters that significantly affect the outcome of MCE was presented. In this method, test vectors are created as inputs to further Monte Carlo simulation (MCSI), and a statistical hypothesis test is applied to the result to determine whether or not each uncertain parameter is influential. The practical effectiveness of the method was demonstrated by its application to the preflight MCE results of the ALFLEX program, and influential parameters affecting touchdown sink rate were identified. The relationship between the number of test vectors and the reliability of hypothesis test was also investigated. Finally, the results of this identification method were compared with those obtained by sensitivity analysis, a conventional method of finding influential parameters. The comparison confirmed the advantages of the presented identification method. With this identification method, Monte Carlo simulation is expected to become a more useful and attractive tool for system evaluation.

References

[1]Shakarian, A., "Application of Monte-Carlo Techniques to the 757/767 Autoland Dispersion Analysis by Simulation," AIAA Paper 83-2193, Aug. 1983.

[2]Hanson, J. M., and Coughlin, D. J., "Ascent, Transition, Entry, and Abort Guidance Algorithm Design for the X-33 Vehicle," AIAA Paper 98-4409, Aug. 1998.

[3]Masciarelli, J. P., Rousseau, S., Fraysse, H., and Perot, E., "An Analytic Aerocapture Guidance Algorithm for the Mars Sample Return Orbiter," AIAA Paper 2000-4116, Aug. 2000.

[4]Stengel, R. F., and Ray, L. R., "Stochastic Robustness of Linear-Time-Invariant Control Systems," *IEEE Transactions on Automatic Control*, Vol. 36, No. 1, 1991, pp. 82–87.

[5]Ray, L. R., and Stengel, R. F., "Application of Stochastic Robustness to Aircraft Control Systems," *Journal of Guidance, Control, and Dynamics*, Vol. 14, No. 6, 1991, pp. 1251–1259.

[6]Ray, L. R., and Stengel, R. F., "A Monte Carlo Approach to the Analysis of Control System Robustness," *Automatica*, Vol. 29, No. 1, 1993, pp. 229–236.

[7]Motoda, T., Miyazawa, Y., Ishikawa, K., and Izumi, T., "Automatic Landing Flight Experiment Flight Simulation Analysis and Flight Testing," *Journal of Spacecraft and Rockets*, Vol. 36, No. 4, 1999, pp. 554–560.

[8]Marrison, C. I., and Stengel, R. F., "Robust Control System Design Using Random Search and Genetic Algorithms," *IEEE Transactions on Automatic Control*, Vol. 42, No. 6, 1997, pp. 835–839.

[9]Marrison, C. I., and Stengel, R. F., "Design of Robust Control Systems for Hypersonic Aircraft," *Journal of Guidance, Control, and Dynamics*, Vol. 21, No. 1, 1998, pp. 58–63.

[10]Stengel, R. F., and Wang, Q., "Searching for Robust Minimal-Order Compensators," *Journal of Dynamic Systems, Measurement and Control*, Vol. 123, No. 2, 2001, pp. 233–236.

[11]Miyazawa, Y., Motoda, T., Izumi, T., and Hata, T., "Longitudinal Landing Control Law for an Autonomous Reentry Vehicle," *Journal of Guidance, Control, and Dynamics*, Vol. 22, No. 6, 1999, pp. 791–800.

[12]Miyazawa, Y., and Motoda, T., "Stochastic Parameter Tuning Applied to Space Vehicle Flight Control Design," *Journal of Guidance, Control, and Dynamics*, Vol. 24, No. 3, 2001, pp. 597–604.

[13]Motoda, T., Stengel, R. F., and Miyazawa, Y., "Robust Control System Design Using Simulated Annealing," *Journal of Guidance, Control, and Dynamics*, Vol. 25, No. 2, 2002, pp. 267–274.

[14]Schubert, W. M., and Stengel, R. F., "Parallel Synthesis of Robust Control Systems," *IEEE Transactions on Control Systems Technology*, Vol. 6, No. 6, 1998, pp. 701–706.

[15]Mooney, C. Z., *Monte Carlo Simulation*, 1st ed., Sage, Thousand Oaks, CA, 1997, pp. 1–5.

[16]Conover, W. J., *Practical Nonparametric Statistics*, 3rd ed., Wiley, New York, 1999, pp. 30, 31, 95–104.

[17]Miyazawa, Y., Nagayasu, M., and Nakayasu, H., "Flight Testing of ALFLEX Guidance, Navigation and Control System," International Council of the Aeronautical Sciences, ICAS Paper 98-1.1.3, Sept. 1998.

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